

Exercise 40

Find the limit.

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sin \pi x}$$

Solution

Rewrite the limit in terms of one that is known.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x}{\sin \pi x} &= \frac{1}{\pi} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{\pi x}{\sin \pi x} \right) = \frac{1}{\pi} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{\pi x}{\sin \pi x} \right) \\ &= \frac{1}{\pi} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\frac{\sin \pi x}{\pi x}} \right) \\ &= \frac{1}{\pi} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\frac{1}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} \right) \\ &= \frac{1}{\pi} (1) \left[\frac{1}{(1)} \right] \\ &= \frac{1}{\pi} \end{aligned}$$